SINGAPORE
MATH KANGAROO CONTEST

Singapore Math Kangaroo Contest 2018

Junior College Contest Paper

Name:_____

School:_____

INSTRUCTIONS:

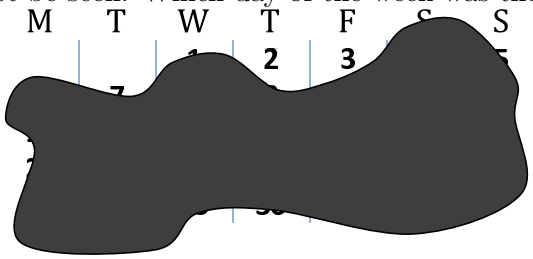
1. Please **DO NOT OPEN** the contest booklet until the Proctor has given permission to start.
2. **Duration**: **1 hour and 30 minutes**
3. There are 30 questions in this paper. Each question scores 3 points in Section A, 4 points in Section B and 5 points in Section C. No points are deducted for Unanswered question. 1 point is deducted for Wrong answer.
4. Shade your answers neatly in the answer entry sheet.
5. PROCTORING: No help should be given to any student in any way during the contest.
6. **No calculators** are allowed.
7. All students must fill and shade in your **Name, Index number, Level and School** in the Answer sheet provided.
8. Students are not allowed to leave the venue within the first hour of the contest and 15 minutes before the end of the contest.
9. Students must show detailed working and transfer their answers to the answer entry sheet.
10. No spare papers can be used in writing this contest. Enough space is provided for your working of each question.
11. Students are not allowed take any answer script, reference materials and contest paper out of the venue.

Rough Working

Section A (Correct – 3 points | Unanswered – 0 points | Wrong – deduct 1 point)

Question 1

The picture shows the calendar of a certain month of the year. Unfortunately some ink fell on the calendar and most of it cannot be seen. Which day of the week was the 27th of that month?



(**A**) Monday (**B**) Wednesday (**C**) Thursday (**D**) Saturday (**E**) Sunday

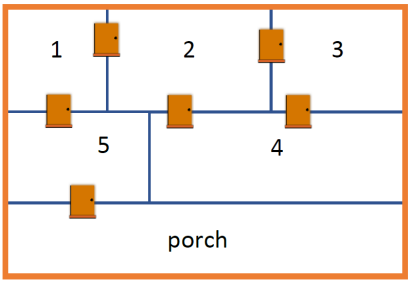
Question 2

Which of the following numerical expressions has the highest value?

(**A**) $2-0\times1+8$ (**B**) $2+0\times1\times8$ (**C**) $2\times0+1\times8$ (**D**) $2\times(0+1+8)$ (**E**) $2\times0+1+8$

Question 3

The figure shows the floor plan of Renate's house. Renate enters her house from the porch and walks through each door exactly once. In which room does she end up?



(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

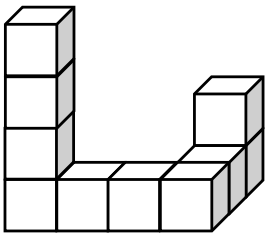
Question 4

Tom has seven stones and a hammer. Every time he hits a stone with the hammer it breaks into exactly five smaller stones. He does this several times. Which of the following numbers could be the number of stones he may end with?

(**A**) 17 (**B**) 20 (**C**) 21 (**D**) 23 (**E**) 25

Question 5

The structure shown below is made of 10 cubes glued together. The shape is dipped into a bucket of paint covering the surface entirely. How many of the cubes will be painted on exactly four of their faces?



(**A**) 6 (**B**) 7 (**C**) 8 (**D**) 9 (**E**) 10

Question 6

The following two statements are true: Some aliens are green, the rest are purple. Green aliens live only on Mars. Therefore, it logically follows that

(**A**) all aliens live on Mars

(**B**) only green aliens live on Mars.

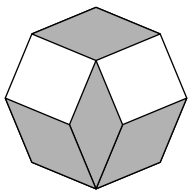
(**C**) some purple aliens live on Venus.

(**D**) all purple aliens live on Venus.

(**E**) no green aliens live on Venus.

Question 7

Four identical rhombusses and two squares are put together to make a regular octogon. What is the measure of the larger angle of each rhombus?



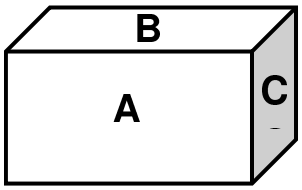
(**A**) 135° (**B**) 140° (**C**) 144° (**D**) 145° (**E**) 150°

Question 8

There are 65 balls in a box. 8 are white and the rest of the balls are black. In one move, at most 5 balls can be taken out of the box. It is not allowed to put any balls back in the box. What is the smallest number of balls must be taken out to ensure that at least one white ball is taken out?

(**A**) 11 (**B**) 12 (**C**) 13 (**D**) 14 (**E**) 15

Question 9
The faces of a rectangular brick have areas A, B and C as shown. What is the volume of the brick?



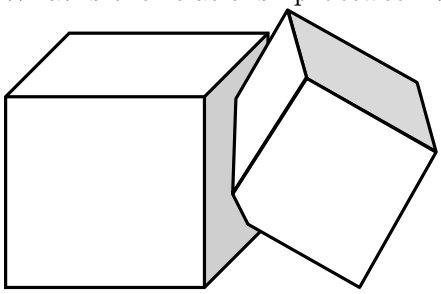
(**A**) ABC (**B**) $\sqrt{ABC}$ (**C**) $\sqrt{AB+BC+CA}$
(**D**) $\sqrt[3]{ABC}$ (**E**) $2(A+B+C)$

Question 10
In how many ways can the number 1001 be written as the sum of two primes?

(**A**) none (**B**) one (**C**) two (**D**) three (**E**) more than three

Section B (Correct – 4 points | Unanswered – 0 points | Wrong – deduct 1 point)

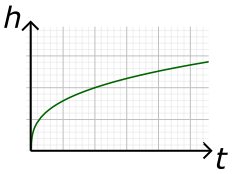
Question 11
Two cubes of volumes V and W intersect. The part of the cube of volume V which is not common to the two cubes is 90% of its volume. The part of the cube of volume W which is not common to the two cubes is 85 % of its volume. What is the relationship between V and W?



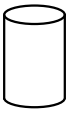
(**A**) $V=\frac{2}{3}W$ (**B**) $V=\frac{3}{2}W$ (**C**) $V=\frac{85}{90}W$
(**D**) $V=\frac{90}{85}W$ (**E**) $V=W$

Question 12

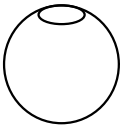
A vase is filled up to the top with water, at a constant rate. The graph shows the height h of the water as a function of time t.

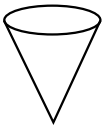


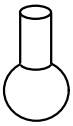
Which of the following could be the shape of the vase?

(**A**) 

(**B**) 

(**C**) 

(**D**) 

(**E**) 

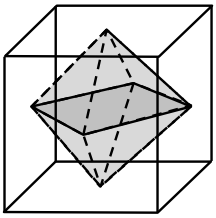
Question 13

$|\sqrt{17}-5|+|\sqrt{17}+5|=$

(**A**) 10 (**B**) $2\sqrt{17}$ (**C**) $\sqrt{34}-10$ (**D**) $10-\sqrt{34}$ (**E**) 0

Question 14

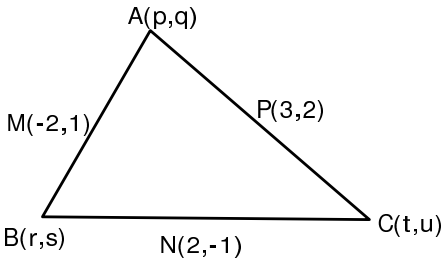
An octahedron is inscribed in a cube of side length 1. The vertices of the octahedron are at the center of the faces of the cube. What is the volume of the octahedron?



(**A**) $\frac{1}{3}$ (**B**) $\frac{1}{4}$ (**C**) $\frac{1}{5}$ (**D**) $\frac{1}{6}$ (**E**) $\frac{1}{8}$

Question 15

The vertices of a triangle are $A(p, q)$, $B(r, s)$ and $C(t, u)$ as shown. The midpoints of the sides of the triangle are the points $M(-2, 1)$, $N(2, -1)$ and $P(3, 2)$. What is the value of $p + q + r + s + t + u$?



(**A**) 2 (**B**) $\frac{5}{2}$ (**C**) 3 (**D**) 5 (**E**) none of these

Question 16

Five predictions were made before the football match between Real Madrid and Manchester United:
1. The game will not end in a draw;
2. Real Madrid will score;
3. Real Madrid will win;
4. Real Madrid will not lose;
5. There will be 3 goals in total.

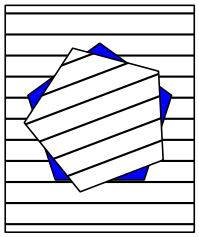
What was the final score of the match between Real Madrid and Manchester United if exactly three of the predictions came true?

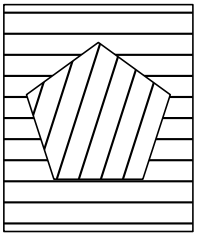
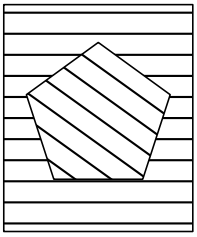
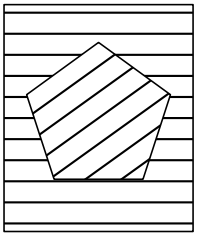
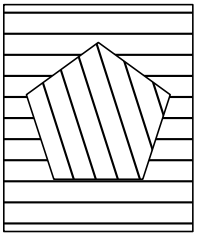
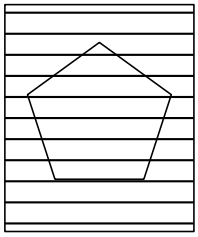
(**A**) 3-0 (**B**) 2-1 (**C**) 0-3 (**D**) 1-2

(**E**) this situation is not possible

Question 17

We cut out a regular pentagon from a lined piece of paper. In each step we rotate the pentagon counter-clockwise around its centre by 21°. The situation after the first step is shown. What will we see when the pentagon first fits back in the hole?



(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

Question 18

Which of these five numbers does not divide $18^{2017} + 18^{2018}$?

(**A**) 8 (**B**) 18 (**C**) 28 (**D**) 38 (**E**) 48

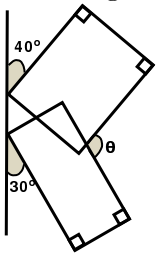
Question 19

Three of the five cards shown are given to Nadia and the rest to Riny. Nadia multiplies the 3 values of her cards and Riny multiplies the 2 values of his cards. It turns out that the sum of the two resulting products is prime. What is the sum of the values in Nadia's cards?

(**A**) 12 (**B**) 13 (**C**) 15 (**D**) 17 (**E**) 18

Question 20

Two rectangles are inclined to the vertical line at angles 40° and 30° as shown. Find the value of θ.

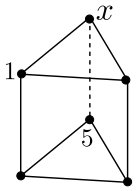


(**A**) 105° (**B**) 120° (**C**) 130° (**D**) 135° (**E**) None of these

Section C (Correct – 5 points | Unanswered – 0 points | Wrong – deduct 1 point)

Question 21

The prism in the picture is formed of two triangles and three squares. The six vertices are numbered from 1 to 6 in such a way that the sum of the four vertices of each square is the same for all three squares. Numbers 1 and 5 are already shown. What number is at the vertex labeled x?



(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 6

(**E**) the situation is impossible

Question 22
Given that m and n are the roots of the equation $x^2 - x - 2018 = 0$, what is the value of $n^2 + m$?

(**A**) 2016 (**B**) 2017 (**C**) 2018 (**D**) 2019 (**E**) 2020

Question 23
Four brothers named A, B, C and D have different heights. They state the following:
A: I am neither the tallest nor the shortest.
B: I am not the shortest.
C: I am the tallest.
D: I am the shortest.
Exactly one of them is lying. Who is the tallest?

(**A**) A (**B**) B (**C**) C (**D**) D

(**E**) We do not have enough information

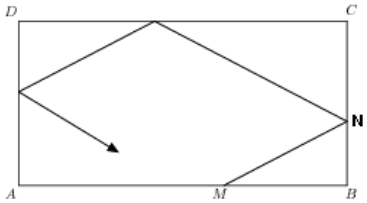
Question 24
Let f be a function such that $f(x + y) = f(x)f(y)$ for all integers x and y. If $f(1) = 1/2$, find the value of $f(0) + f(1) + f(2) + f(3)$.

(**A**) 1/8 (**B**) 3/2 (**C**) 5/2 (**D**) 15/8 (**E**) 6

v
Question 25
A quadratic function $f(x) = x^2 + px + q$ is such that its graph intersects the x-axis and the y-axis in three different points. The circle through these three points intersects the graph of f in a fourth point. What are the coordinates of this fourth point?

(**A**) $(0, -q)$ (**B**) (p, q) (**C**) $(-p, q)$ (**D**) $\left(-\frac{q}{p}, \frac{q^2}{p^2}\right)$ (**E**) $(1, p + q + 1)$

Question 26
We are given a rectangular billiard table with sides of length $3m$ and $2m$. A ball is shot from the point M on one of the longer sides. It reflects once on every other side as shown. At what distance from point A will it hit the initial side if $BM = 1.2m$ and $BN = 0.8m$?



(**A**) $1.2m$ (**B**) $1.5m$ (**C**) $2m$ (**D**) $0.8m$ (**E**) $1.8m$

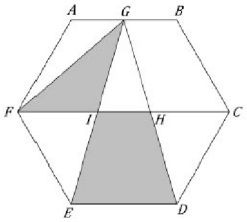
Question 27

Find the number of real solutions in the equation: $\big||4^x - 3| - 2\big| = 1$

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

Question 28

$ABCDEF$ is a regular hexagon. G is the midpoint of AB. H and I are the points of intersection of the segments GD and GE with FC respectively. What is the ratio of the area of the triangle GIF to the area of the trapezoid $IHDE$?



(**A**) $\frac{1}{2}$ (**B**) $\frac{1}{3}$ (**C**) $\frac{1}{4}$ (**D**) $\frac{\sqrt{3}}{3}$ (**E**) $\frac{\sqrt{3}}{4}$

Question 29

There are 40 % more girls than boys in a class. How many pupils are in this class if the probability that a two-person delegation selected at random consists of a girl and a boy equals $\frac{1}{2}$?

(**A**) 20 (**B**) 24 (**C**) 36 (**D**) 38

(**E**) This situation is not possible.

Question 30

Archimedes calculated 15!. The result is written on the board. Unfortunatelly two of the figures, the second and the tenth, are not visible. Which are these two figures?

$$1\blacksquare 0767436\blacksquare 000$$

(**A**) 2 and 0 (**B**) 4 and 8 (**C**) 7 and 4

(**D**) 9 and 2 (**E**) 3 and 8

Rough Working

Rough Working