Singapore Math Kangaroo Contest 2016

Junior College Contest Paper

Name:

School:

INSTRUCTIONS:

1. Please **DO NOT OPEN** the contest booklet until the Proctor has given permission to start.

2. **TIME** : **1 hour and 30 minutes**

3. There are 30 questions in this paper. Each question scores 3 points in Section A, 4 points in Section B and 5 points in Section C. No points are deducted for Unanswered question. 1 point is deducted for Wrong answer.

4. Shade your answers neatly in the answer entry sheet.

5. PROCTORING : No one may help any student in any way during the contest.

6. **No calculators** are allowed.

7. All students must fill and shade in your **Name, Index number, Level and School** in the Answer sheet

8. MINIMUM TIME: Students must stay in the exam hall for at least 1 hour and 15 minutes.

9. Students must show detailed working and transfer answers to the answer entry sheet.

10. No spare papers can be used in writing this contest. Enough space is provided for your working of each question.

11. You must return this contest paper to the proctor.

Rough Working

Section A (Correct – 3 points | Unanswered – 0 points | Wrong – deduct 1 point)

1. The sum of the ages of Tom and John is 23, the sum of the ages of John and Alex is 24 and the sum of the ages of Tom and Alex is 25. What is the age of the oldest among them?

(**A**) 10   (**B**) 11   (**C**) 12   (**D**) 13   (**E**) 14

2. The value of $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000}$ is

(**A**) $\frac{3}{111}$ (**B**) $\frac{111}{1110}$ (**C**) $\frac{111}{1000}$ (**D**) $\frac{3}{1000}$ (**E**) $\frac{3}{1110}$

**3.** Maria wants to build a bridge across a river. The shortest possible bridge from each point on one shore is always of the same length. Which of these pictures cannot be a picture of the river?

(**A**)   (**B**)   (**C**)

(**D**)   (**E**)

4. How many integers are greater than 2015×2017 but less than 2016×2016?

(**A**) 0   (**B**) 1   (**C**) 2015   (**D**) 2016   (**E**) 2017

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5. A set of points forms a picture of a kangaroo in the xy-plane as shown. For each point the x and y coordinates are swapped. What is the result?

(**A**)

(**B**)

(**C**)

(**D**)

(**E**)

2
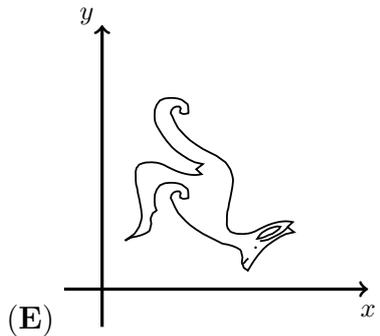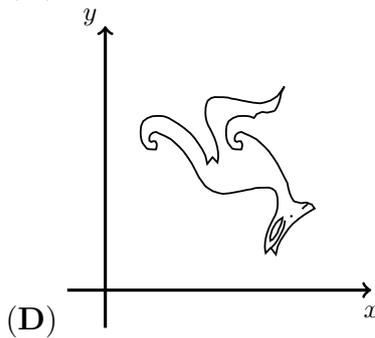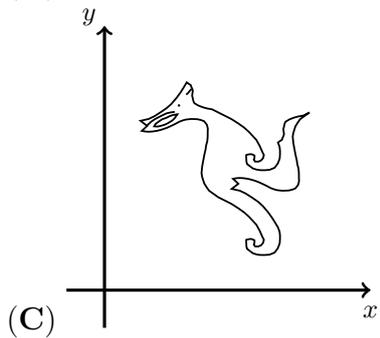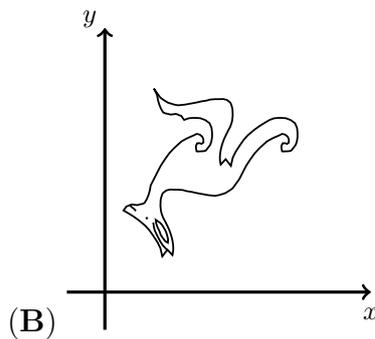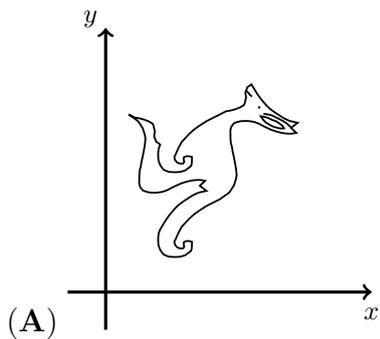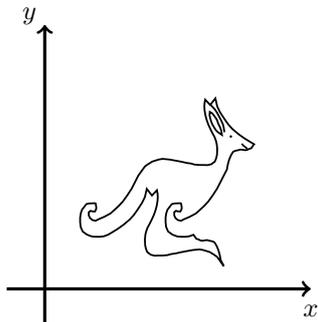
6. What is the smallest number of planes needed to enclose a bounded part in three-dimensional space?

(**A**) 3   (**B**) 4   (**C**) 5   (**D**) 6   (**E**) 7

7. Diana wants to write nine integers into the circles on the diagram so that, for the eight small triangles whose vertices are joined by segments the sums of the numbers in their vertices are identical. What is the largest number of different integers she can use?
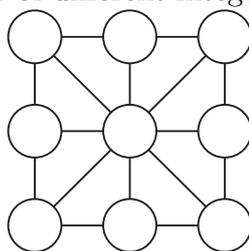
(**A**) 1   (**B**) 2   (**C**) 3   (**D**) 5   (**E**) 8

8. The rectangles S_1 and S_2 in the picture have the same area. Determine the ratio $\frac{x}{y}$.

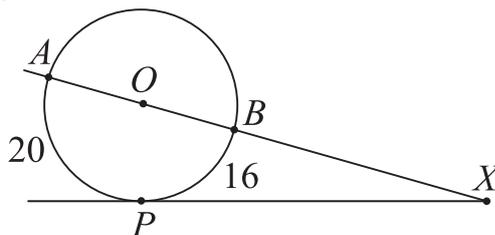(**A**) 1   (**B**) $\frac{3}{2}$ (**C**) $\frac{4}{3}$ (**D**) $\frac{7}{4}$ (**E**) $\frac{8}{5}$

3
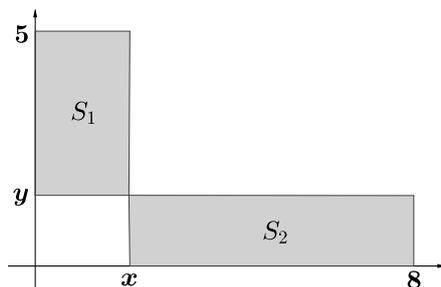
9. If $x^2 - 4x + 2 = 0$, then $x + \frac{2}{x}$ is equal to

(**A**) -4   (**B**) -2   (**C**) 0   (**D**) 2   (**E**) 4

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10. The lengths of arc AP and arc BP in the figure are 20 and 16, respectively. Then the value of the angle $\angle AXP$ equals

(**A**) 30°   (**B**) 24°   (**C**) 18°   (**D**) 15°   (**E**) 10°

Section B (Correct – 4 points | Unanswered – 0 points | Wrong – deduct 1 point)

11. a, b, c, d are positive integers satisfying $a + 2 = b - 2 = c \times 2 = d : 2$. Which is the largest of the four numbers a, b, c and d?

(**A**) a (**B**) b (**C**) c

(**D**) d (**E**) This is not uniquely determined.

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12. In this pyramid of numbers each upper field is the product of the two fields directly underneath. Which of the following numbers cannot appear in the top field, if the three bottom fields only contain integers greater than 1?
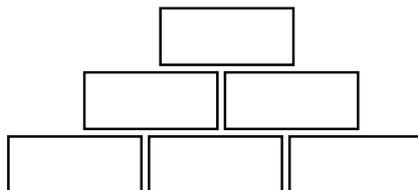
(**A**) 56   (**B**) 84   (**C**) 90   (**D**) 105   (**E**) 220

13. What is x_4, if $x_1 = 2$ and $x_{n+1} = x_n^{x_n}$ for $n \geq 1$?

(**A**) 2^{2^3} (**B**) 2^{2^4} (**C**) $2^{2^{11}}$ (**D**) $2^{2^{16}}$ (**E**) $2^{2^{768}}$

14. In rectangle $ABCD$, the length of the side $\overline{BC}$ is half the length of the diagonal $\overline{AC}$. Let M be a point on CD such that $|\overline{AM}| = |\overline{MC}|$. What is the value of the angle $\angle CAM$?

(**A**) $12.5°$ (**B**) $15°$ (**C**) $27.5°$ (**D**) $42.5°$ (**E**) $21.5°$

15. Diana cut up a rectangle of area 2016 into 56 equal squares. The lengths of the sides of the rectangle and of the squares are integers. For how many different rectangles is it possible for her to do this? (Two rectangles that can be obtained from one another by rotating 90 degrees are considered the same.)

(**A**) 2   (**B**) 4   (**C**) 6   (**D**) 8   (**E**) 0

16. On Fantasy Island every citizen is either a Knight (who always speaks the truth) or a Knave (who always lies). During your travels on the island you meet 7 people sitting around a bonfire. They all tell you "I'm sitting between two Knaves!" How many Knaves are there?

(**A**) 3   (**B**) 4   (**C**) 5   (**D**) 6   (**E**) 7

17. The equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ both have real roots. It is known that the sum of squares of the roots of the first equation is equal to the sum of squares of the roots of the second one, and $a \neq b$. Then $a + b$ is equal to

(**A**) 0   (**B**) -2   (**C**) 4   (**D**) -4   (**E**) 6

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18. If the perimeter of the square in the figure equals 4 then the perimeter of the equilateral triangle is equal to
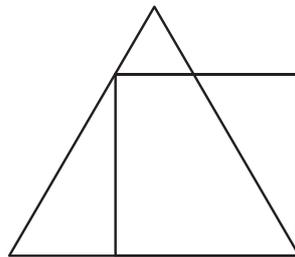

(**A**) 4   (**B**) $3 + \sqrt{3}$ (**C**) 3   (**D**) $3 + \sqrt{2}$ (**E**) $4 + \sqrt{3}$

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19. Each of ten points in the figure is labelled with either 0 or 1 or 2. It is known that the sum of numbers in the vertices of any white triangle is divisible by 3, while the sum of numbers in the vertices of any black triangle is not divisible by 3. Three points are labelled as shown in the figure. What numbers can be used to label the central point?
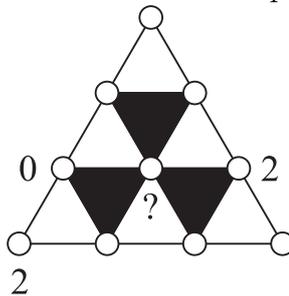
(**A**) Only 0.   (**B**) Only 1.   (**C**) Only 2.   (**D**) Only 0 and 1. (**E**) Either 0 or 1 or 2.

20. Ben draws five points A, B, C, D and E on a circle as well as the tangent to the circle at A, such that all five angles marked with x are equal. (Note that the drawing is not to scale.) Find the angle $\angle ABD$?
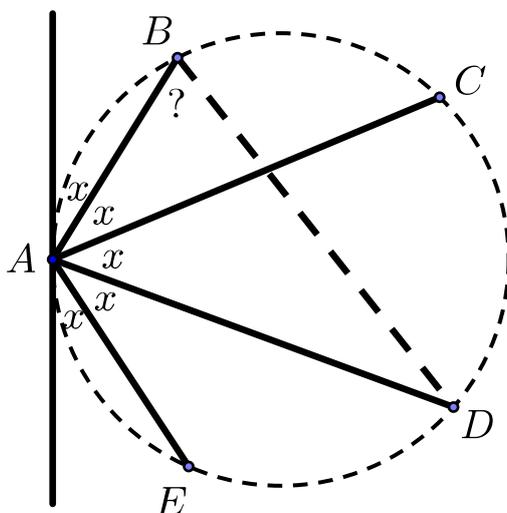
(**A**) 66°   (**B**) 70.5°   (**C**) 72°   (**D**) 75°   (**E**) 77.5°

Section C (Correct – 5 points | Unanswered – 0 points | Wrong – deduct 1 point)

**21.** Determine the number of real solutions to theequation

$$\left(x^2 - 4x + 5\right)^{x^2+x-30} = 1$$

(**A**) 1   (**B**) 2   (**C**) 3   (**D**) 4   (**E**) 5

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22. A quadrilateral contains an inscribed circle (i.e. a circle tangent to the four sides of the quadrilateral). The ratio of the perimeter of the quadrilateral to the perimeter of the circle is 4:3. Then the ratio of the area of the quadrilateral to the area of the circle is

(**A**) $4 : \pi$ (**B**) $3\sqrt{2} : \pi$ (**C**) $16 : 9$ (**D**) $\pi : 3$ (**E**) $4 : 3$

23. How many quadratic functions in x have its graph passing through at least 3 of the marked points?
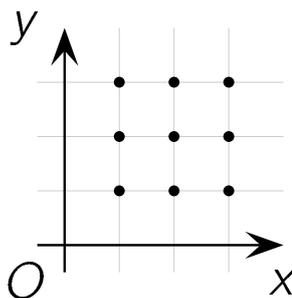
(**A**) 6 (**B**) 15 (**C**) 19 (**D**) 22 (**E**) 27

24. In a right-angled triangle ABC (right angle at A), the bisectors of the acute angles intersect at point P. If the distance from P to the hypotenuse is $\sqrt{8}$, what is the distance from P to A?

(**A**) 8 (**B**) 3 (**C**) $\sqrt{10}$ (**D**) $\sqrt{12}$ (**E**) 4

25. Three three-digits numbers are formed using the digits from 1 to 9 (each digit is used exactly once). Which of the following numbers could not be equal to the sum of these three numbers?

(**A**) 1500 (**B**) 1503 (**C**) 1512 (**D**) 1521 (**E**) 1575

26. A cube is dissected into 6 pyramids by connecting a given point in the interior of the cube with each vertex of the cube. The volumes of five of these pyramids are 2, 5, 10, 11 and 14. What is the volume of the sixth pyramid?

(**A**) 1 (**B**) 4 (**C**) 6 (**D**) 9 (**E**) 12

27. A rectangular strip $ABCD$ of paper 5 cm wide and 50 cm long is white on one side and grey on the other. Folding the strip, Cristina makes the vertex B coincide with the midpoint M of the side CD. Folding again, she makes the vertex D coincide with the midpoint N of the side AB. What is the area (in cm^2) of the visible white part of the strip in the last picture?
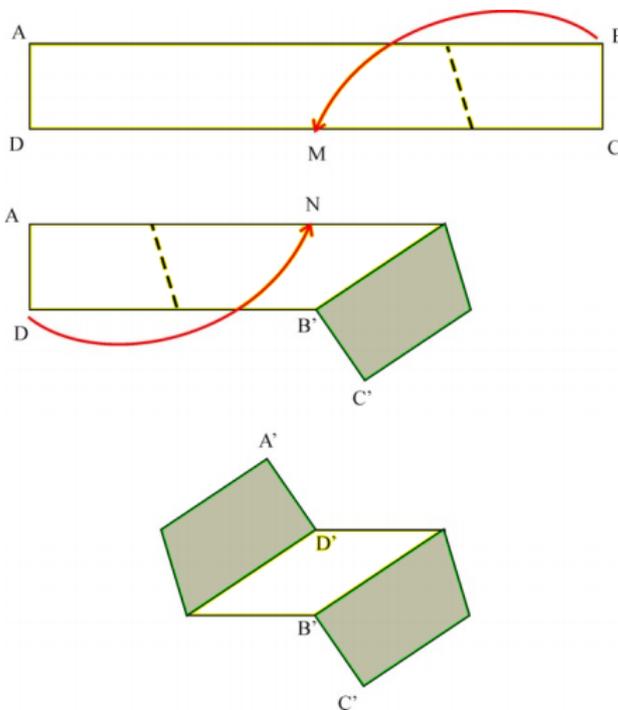

(**A**) 50   (**B**) 60   (**C**) 62.5   (**D**) 100   (**E**) 125

28. Ann chose a positive integer n and wrote down the sum of all positive integers from 1 to n. A prime number p divides the sum, but not any of the summands. Which of the following could be $n + p$?

(**A**) 217   (**B**) 221   (**C**) 229

(**D**) 245   (**E**) 269

29. Consider a 5×5 square divided into 25 cells. Initially all its cells are white. In each move it is allowed to change the color of any three consecutive cells in a row or in a column to the opposite color (i.e. white cells become black and black ones become white). What is the smallest possible number of moves needed to obtain the chessboard coloring shown in the figure?
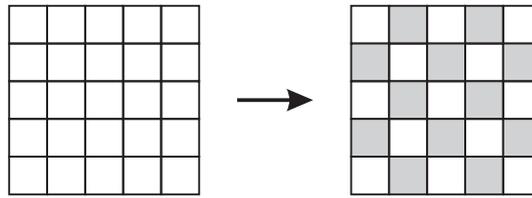
(**A**) less than 10   (**B**) 10   (**C**) 12   (**D**) 14   (**E**) 20

30. The positive integer N has exactly six distinct positive divisors including 1 and N. The product of five of these divisors is 648. Which one of the following is the sixth divisor of N?

(**A**) 4   (**B**) 8   (**C**) 9   (**D**) 12   (**E**) 24

END OF PAPER

Rough Working

Rough Working