Singapore Math Kangaroo Contest 2017

Junior College Contest Paper

Name:_____

School:_____

INSTRUCTIONS:

1. Please **DO NOT OPEN** the contest booklet until the Proctor has given permission to start.
2. **TIME** : **1 hour and 30 minutes**
3. There are 30 questions in this paper. 3 points, 4 points and 5 points will be awarded for each correct question in Section A, B and C respectively. Points are not deducted for unanswered questions. 1 point will be deducted for every wrong answer.
4. Shade your answers neatly in the answer entry sheet.
5. PROCTORING : No one may help any student in any way during the contest.
6. **Calculators** are not allowed.
7. Students are required to shade in your **Name, Index number, Level and School** in the Answer sheet provided.
8. MINIMUM TIME: Students must stay in the exam hall for at least 1 hour and 15 minutes.
9. Students must show detailed working and transfer their final answers to the answer entry sheet.
10. Any additional papers is not allowed for this contest. Sufficient space will be provided in the contest paper.
11. You must return this contest paper to the proctor.

Rough Working

Section A (Correct – 3 points | Unanswered – 0 points | Wrong – deduct 1 point)

Question 1

$$\frac{20 \times 17}{2+0+1+7} =$$

(**A**) 3.4 (**B**) 17 (**C**) 34 (**D**) 201.7 (**E**) 340

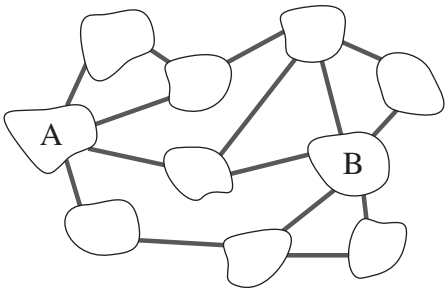
Question 2

Tom built a figurine of his brother using the ratio of 1:87. If the height of the figurine is 2cm, what is actual the height of his brother?

(**A**) 1.74 m (**B**) 1.62 m (**C**) 1.86 m (**D**) 1.94 m (**E**) 1.70 m

Question 3

In the figure below, we can see 10 islands that are connected by 15 bridges. If all bridges are open, what is the smallest number of bridges that must be closed in order to stop the traffic between A and B?



(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

Question 4

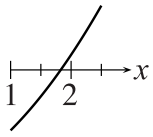
It is given that 75% of a is equal to 40% of b. Which option below is always true?

(**A**) $15a = 8b$ (**B**) $7a = 8b$ (**C**) $3a = 2b$ (**D**) $5a = 12b$ (**E**) $8a = 15b$

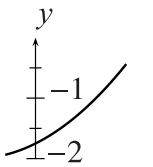
Question 5

Four of the following five clippings are part of the same graph of the quadratic function. Which clipping is not part of this graph?

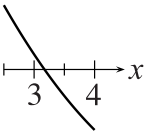
(**A**)



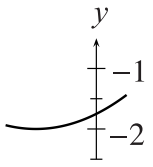
(**B**)



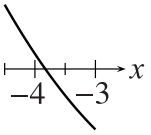
(**C**)



(**D**)

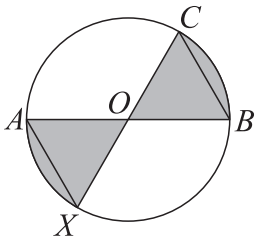


(**E**)



Question 6

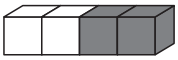
Given a circle with center O with diameters AB and CX such that $OB = BC$. What fraction of the area of the circle is the shaded?

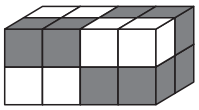
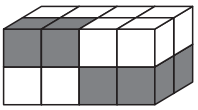
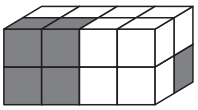
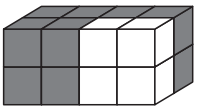
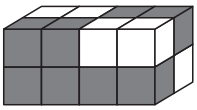


(**A**) $\frac{2}{5}$ (**B**) $\frac{1}{3}$ (**C**) $\frac{2}{7}$ (**D**) $\frac{3}{8}$ (**E**) $\frac{4}{11}$

Question 7

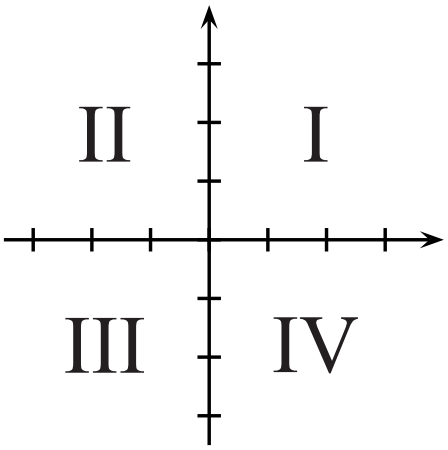
A bar consists of 2 white and 2 grey cubes glued together as shown in the picture below. Which figure can be built from 4 such bars?



(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

Question 8

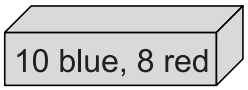
Which quadrant contains no points on the graph of the linear function $f(x) = -3.5x + 7$?

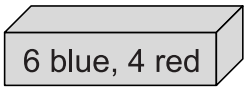


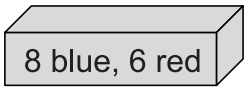
(**A**) I (**B**) II (**C**) III

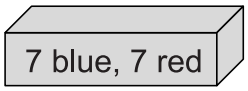
(**D**) IV (**E**) All quadrants contain points.

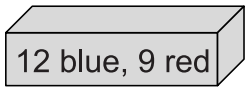
Question 9
Each of the following five boxes is filled with red and blue balls as labeled. Ben wants to take one ball out of the boxes without looking. From which box should he take the ball to have the highest probability of getting a blue ball?

(**A**) 

(**B**) 

(**C**) 

(**D**) 

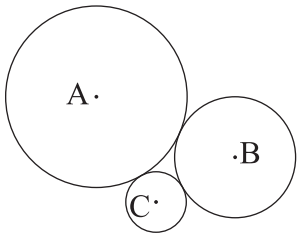
(**E**) 

Question 10
Find the graph which has the most number of common points with the graph of the function $f(x) = x$?

(**A**) $g_1(x) = x^2$ (**B**) $g_2(x) = x^3$ (**C**) $g_3(x) = x^4$ (**D**) $g_4(x) = -x^4$ (**E**) $g_5(x) = -x$

Section B (Correct – 4 points | Unanswered – 0 points | Wrong – deduct 1 point)

Question 11
Three mutually tangent circles with centres A, B, C have the radii 3, 2 and 1, respectively. What is the area of the triangle ABC?



(**A**) 6 (**B**) $4\sqrt{3}$ (**C**) $3\sqrt{2}$ (**D**) 9 (**E**) $2\sqrt{6}$

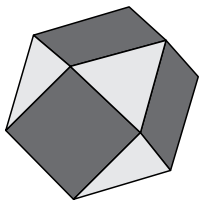
Question 12
The positive number p is less than 1, and the number q is greater than 1. Which one of the following numbers is the largest?

(**A**) $p \cdot q$ (**B**) $p + q$ (**C**) $\frac{p}{q}$ (**D**) p (**E**) q

Question 13
Two right cylinders A and B have the same volume. If the radius of the base of B is 10 % larger than the base of A. How much larger is the height of A than the height of B?

(**A**) 5 % (**B**) 10 % (**C**) 11 % (**D**) 20 % (**E**) 21 %

Question 14
Each face of the polyhedron shown below is either triangle or a square. Each square is surrounded by 4 triangles, while each triangle is surrounded by 3 squares. If there are 6 squares in total, how many triangles are there?



(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 9

Question 15

We have four tetrahedral dice, perfectly balanced, with their faces numbered 2, 0, 1 and 7. If we roll all four of these dice, what is the probability that we can form the number 2017 using exactly one of the three visible numbers from each die?

(**A**) $\frac{1}{256}$ (**B**) $\frac{63}{64}$ (**C**) $\frac{81}{256}$ (**D**) $\frac{3}{32}$ (**E**) $\frac{29}{32}$

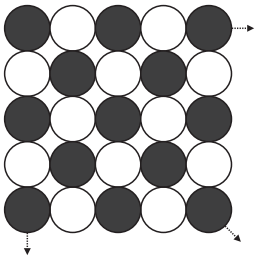
Question 16

The polynomial $5x^3 + ax^2 + bx + 24$ has integer coefficients a and b. Which of the following is certainly not a root of the polynomial?

(**A**) 1 (**B**) -1 (**C**) 3 (**D**) 5 (**E**) 6

Question 17

Julia has 2017 chips. 1009 of them are black and the rest are white. She placed them in a square pattern as shown in the figure below. How many chips of each colour are left after she has formed the largest possible square using her chips?



(**A**) None
(**B**) 40 of each
(**C**) 40 black ones and 41 white ones
(**D**) 41 of each
(**E**) 40 white ones and 41 black ones

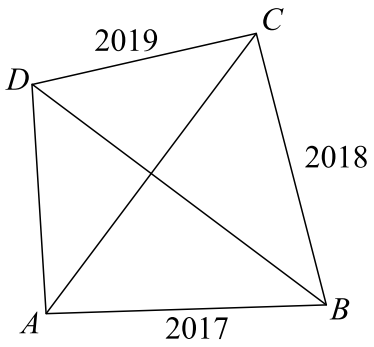
Question 18

Two consecutive numbers are such that the sums of their digits in each of them are multiples of 7. How many digits does the smaller number have?

(**A**) 3 (**B**) 4 (**C**) 5 (**D**) 6 (**E**) 7

Question 19

In a convex quadrilateral $ABCD$, the diagonals are perpendicular to each other. The sides have lengths $|AB| = 2017$, $|BC| = 2018$ and $|CD| = 2019$. What is the length of AD?



(**A**) 2016 (**B**) 2018 (**C**) $\sqrt{2020^2 - 4}$ (**D**) $\sqrt{2018^2 + 2}$ (**E**) 2020

Question 20

Taylor attempts to be a good little Kangaroo, but lying is too much fun. Therefore, every third statement she says is a lie while the rest are true. (Sometimes she starts with a lie and sometimes with one or two true statements.)

Taylor thinks of a 2-digit number and tells her friend about it:

"One of its digits is a 2."

"It is larger than 50."

"It is an even number."

"It is less than 30."

"It is divisible by three."

"One of its digits is a 7."

What is the sum of the digits in Taylor's number?

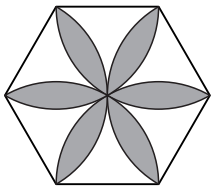
(**A**) 9 (**B**) 12 (**C**) 13 (**D**) 15 (**E**) 17

Section C (Correct – 5 points | Unanswered – 0 points | Wrong – deduct 1 point)

Question 21
When you remove the last digit in a positive integer, it is equal to 1/14 of the original number. How many such positive integers are there?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

Question 22
The picture shows a regular hexagon with each side lengths equal to 1. The flower was constructed with sectors of circles of radius 1 and centers on the vertices of the hexagon. What is the area of the flower?



(**A**) $\frac{\pi}{2}$ (**B**) $\frac{2\pi}{3}$ (**C**) $2\sqrt{3}-\pi$ (**D**) $\frac{\pi}{2}+\sqrt{3}$ (**E**) $2\pi-3\sqrt{3}$

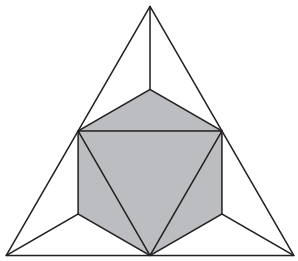
Question 23

Consider the sequence a_n with $a_1 = 2017$ and $a_{n+1} = \dfrac{a_n - 1}{a_n}$. What is the value of a_{2017}?

(**A**) -2017 (**B**) $\dfrac{-1}{2016}$ (**C**) $\dfrac{2016}{2017}$ (**D**) 1 (**E**) 2017

Question 24

Consider a regular tetrahedron. Its four corners are cut off by four planes, each passing through the midpoints of three adjacent edges as shown in the figure below. What is the ratio of the volume of the resulting solid to the volume of the original tetrahedron?



(**A**) $\dfrac{4}{5}$ (**B**) $\dfrac{3}{4}$ (**C**) $\dfrac{2}{3}$ (**D**) $\dfrac{1}{2}$ (**E**) $\dfrac{1}{3}$

Question 25

The lengths of the three sides of a right-angled triangle add up to 18. The squares of the lengths of the sides add up to 128. What is the area of the triangle?

(**A**) 18 (**B**) 16 (**C**) 12 (**D**) 10 (**E**) 9

Question 26

You are given 5 boxes, 5 black and 5 white balls. You choose how to put the balls in the boxes (each box has to contain at least one ball). Your opponent comes and draws one ball from one box of his choice and he wins if he draws a white ball. Otherwise, you win. How should you arrange the balls in the boxes to have the best chance to win?

(**A**) You put one white and one black ball in each box.

(**B**) You arrange all the black balls in three boxes, and all the white balls in two boxes.

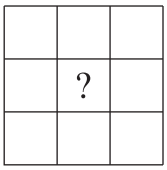
(**C**) You arrange all the black balls in four boxes, and all the white balls in one box.

(**D**) You put one black ball in every box, and add all the white balls in one box.

(**E**) You put one white ball in every box, and add all the black balls in one box.

Question 27

Nine integers are written in the cells of a 3×3 table. The sum of the nine numbers is equal to 500. It is known that the numbers in any two neighboring cells (with a common side) differ by 1. What is the number in the central cell?



(**A**) 50 (**B**) 54 (**C**) 55 (**D**) 56 (**E**) 57

Question 28

If $|x| + x + y = 5$ and $x + |y| - y = 10$ what is the value of $x + y$?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4

(**E**) 5

Question 29

How many three-digit positive integers ABC are there such that $(A+B)^C$ is a three-digit integer and an integer power of 2?

(**A**) 15 (**B**) 16 (**C**) 18 (**D**) 20 (**E**) 21

Question 30

Each of the 2017 people living on an island is either a liar (and always lies) or a truth-teller (and always tells the truth). More than one thousand of them take part in a banquet, all sitting together at a round table. Each of them says: "Of the two people beside me, one is a liar and the other one a truth-teller." How many truth-tellers are there on the island at most?

(**A**) 1683 (**B**) 668 (**C**) 670 (**D**) 1344 (**E**) 1343

Rough Working

Rough Working